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Endowment structures, industrial dynamics, and economic growth

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ABSTRACT

Motivated by four stylized facts about industry dynamics, we propose a theory of endowment-driven structural change by developing a tractable growth model with infinite industries. The aggregate economy in the model still follows the Kaldor facts, but the composition of the underlying industries changes endogenously over time. Each industry exhibits a hump-shaped life cycle: as capital reaches a certain threshold level, a new industry appears, prospers, and then declines, to be gradually replaced by a more capital-intensive industry, *ad infinitum*. Analytical solutions are obtained to characterize the life cycle of each industry and the perpetual structural change.

1. Introduction

Sustainable economic growth relies on the healthy development of underlying industries. Enormous research progress has been made on industrial dynamics, yet many important aspects still remain imperfectly understood within the context of economic growth, especially at the high-digit disaggregated industry level. Consider, for example, the automobile industry and the apparel industry. How different are the evolution patterns of two industries along the growth path of the whole economy? Which industry should be expected to expand or decline earlier than the other and why? How long, if at all, does a leading industry maintain its predominant position? What fundamental forces drive these dynamics? What is the relationship between individual industrial dynamics and aggregate GDP growth? These questions are interesting to economists, policy makers, and private investors.

The goal of this paper is to shed light on these issues by studying the dynamics of all high-digit industries simultaneously within a growth framework. We establish four stylized facts about industrial dynamics using the NBER-CES data set of the US manufacturing sector, which covers 473 industries at the 6-digit NAICS level from 1958 to 2005. First, there exists tremendous cross-industry heterogeneity both in capital intensities and productivities. Second, the value-added share (or the employment share) of an industry typically exhibits a hump-shaped life cycle, that is, an industry first expands, reaches a peak, and eventually declines. Third, a more capital-intensive industry reaches its peak later. Fourth, the further away an industry's factor intensity deviates from the factor endowment of the economy, the smaller is the employment share (or output share) of this industry in the aggregate economy, which may be called the *congruence fact*. Similar patterns are also found in the UNIDO data set, which covers 166 countries from 1963 to 2009 at the two-digit level (23 sectors). In fact,

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documentation and analyses of a subset of the above-mentioned patterns of industrial dynamics can be dated at least back to the 1960s. For example, Chenery and Taylor (1968) show that the major products in the manufacturing sector gradually shift from the labor-intensive ones to more capital-intensive ones as an economy develops.

Motivated and guided by these four stylized facts, we develop a tractable endogenous growth model with multiple industries to explain these observed patterns of industrial dynamics. In light of Fact 3 and Fact 4, together with theoretical arguments for the importance of capital accumulation in the growth literature, we revisit the role of the endowment structure, measured by the aggregate capital–labor ratio, in both determining the production structure and driving the industrial dynamics.[1] More specifically, our model is constructed to explain the second, third and fourth stylized facts simultaneously by taking the first fact as given.

To understand how endowment structure may affect different industries, one may appeal to the Rybczynski Theorem, which states that in a static model with two goods and two factors, the capital-intensive sector expands while the labor-intensive sector shrinks as the economy becomes more capital abundant. This result holds for both an autarky and an open economy. The intuition is that the rental wage ratio declines as capital becomes more abundant, so this change in the endowment structure disproportionately favors the production of the capital-intensive good, holding other things constant.

This logic is straightforward and well known, but it is not obvious whether capital accumulation itself can simultaneously explain Facts 2 through 4 in a growth model with a large number of industries. Recall that the Rybczynski Theorem says almost nothing about what happens when there are more than two goods (Feenstra, 2003). However, for our purpose, a model with many (ideally infinite) industries is indispensable, not only because it is much more realistic at a high-digit industry level, but also because the multilateral interactions among different industries could be potentially important for industrial dynamics.

More important technical challenges for such a formal model come from the dynamic part of the problem in the presence of many industries, each of which evolves nonlinearly. More precisely, first, the model predictions should be not only consistent with the aforementioned stylized facts at the disaggregated industry level, but also consistent with the Kaldor facts at the aggregate level.[2] Second, to keep track of the life-cycle dynamics of each industry along the whole path of aggregate growth, we must fully characterize transitional dynamics, which is well-recognized to be difficult even for a two-sector model, but now there exist infinite industries with an infinite time horizon.[3] Third, it turns out that endogenous structural change in the underlying industries eventually forces us to characterize a Hamiltonian system with endogenously switching state equations, as will be clearer in the model.

All these technical challenges create important obstacles for theoretical research on industrial dynamics. Fortunately, a desirable feature of our model is precisely its tractability: closed-form solutions are obtained to fully characterize the whole process of the hump-shaped industrial dynamics for each of the infinite industries along the aggregate growth path. The model predictions are qualitatively consistent with all the stylized facts about the industrial dynamics at the micro-industry level and the Kaldor facts at the aggregate level.

We first develop a static model with infinite industries (or goods, interchangeably) and two factors (labor and capital). With a general CES production function for the final commodity, we obtain a version of the *Generalized Rybczynski Theorem*: for any given endowment of capital and labor, there exists a cutoff industry such that, when the capital endowment increases, the output will increase in every industry that is more capital intensive than this cutoff industry, while the output in all the industries that are less capital intensive than this cutoff industry will decrease. Moreover, the cutoff industry moves toward the more capital-intensive direction as the capital endowment increases. As a special case, when the CES substitution elasticity is infinity, generically only two industries are active in equilibrium and the capital–labor ratios of the active industries are the closest to the capital–labor ratio of the economy. This result is consistent with the congruence fact. The model implies that the structures of underlying industries are endogenously different at different stages of economic development.

Then the model is extended to a dynamic environment where capital accumulates endogenously. The dynamic decision is decomposed into two steps. First, the social planner optimizes the intertemporal allocation of capital for the production of consumption goods, which determines the evolution of the endowment structure. Then, at each time point the resource allocation across different industries is determined by the capital and labor endowments in the same way as the static model. Endogenous changes in the industry composition of an economy translate into different functional forms of the endogenous aggregate production function and the capital accumulation function; therefore, one must solve a Hamiltonian system with endogenously switching state equations because of the endogenous structural change. When the CES substitution elasticity is infinity (linear case), it is shown that there always exists a unique aggregate growth path with a constant growth rate, along which the underlying industries shift over time by following a hump-shaped pattern with the more capital-intensive ones reaching a peak later (Facts 2 and 3). We analytically characterize the life-cycle dynamics of each industry and also show that the model predictions are empirically consistent with the data.

[1] In his Marshall Lectures, Lin (2009) proposes that many development issues such as growth, inequality, and industrial policies can all be better understood by analyzing the congruence of the industrial structure with the comparative advantages determined by the endowment structure and its change. There, the endowment structure is more formally defined as the composition of the production factors (including labor, human capital, physical capital, land and other natural resources) as well as the soft and hard infrastructures.

[2] Kaldor facts refer to the relative constancy of the growth rate of total output, the capital–output ratio, the real interest rate, and the share of labor income in GDP.

[3] See King and Rebelo (1993), Mulligan and Sala-i-Martin (1993), Bond et al. (2003), and Mehlum (2005).

Our paper is most closely related to the growth literature on structural change, which studies the resource allocation across different sectors as an economy grows (see Herrendorf et al., 2014 for a recent survey). This literature mainly tries to match the Kuznets facts, namely, the agriculture share in GDP has a secular decline, the industry (manufacturing) share demonstrates a hump shape, and the service share increases. However, such sectors are too aggregated to address questions such as those raised in the first paragraph. Insufficient effort is devoted to reconciling Kaldor facts with the aforementioned stylized facts about industrial dynamics at much more disaggregated levels.

The empirical contribution of this paper to the literature is that we formally establish four stylized facts about industry dynamics. In addition, our paper makes two theoretical contributions. First, we develop a highly tractable growth model with infinite industries to fully characterize the industrial dynamics, which are qualitatively consistent with the four motivating facts. Second, we show how capital accumulation serves as a new and important mechanism that drives the structural change. The existing literature mainly discusses two mechanisms of structural change in autarky. One is the preference-driven mechanism, in which the demand for different goods shift asymmetrically as income increases due to the non-homothetic preferences.[4] The second mechanism is that unbalanced productivity growth rates across sectors drive resource reallocation.[5]

Unlike these two mechanisms, we propose that improvement of endowment structure (capital accumulation) itself is a new and fundamental mechanism that drives industrial dynamics, which is referred to as endowment-driven structural change. To highlight the theoretical sufficiency and distinction of this new mechanism, we assume a homothetic preference to shut down the preference-driven mechanism. Conceptually, it is not obvious how to order industries according to their income demand elasticities at a highly disaggregated level. It is also assumed in the benchmark model that productivity is constant over time in all the industries to shut down the productivity-driven mechanism. Instead, our model, as motivated by Fact 1, assumes that industries differ in their capital intensities, deviating from the standard assumption that different sectors have equal capital intensity, including models without capital.[6]

An important exception is Acemoglu and Guerrieri (2008), who study structural change in a two-sector growth model with different capital intensities, but their model does not aim to explain or generate the repetitive hump-shaped industrial dynamics because the life cycle of each sector is truncated. Instead, their analytical focus is on the *asymptotic* aggregate growth rate in the long run, by which time one industry dominates the economy in terms of employment share and structural change virtually ends. In contrast, our model has infinite sectors so the structural change is endless and this setting allows us to analyze the complete life-cycle dynamics of *every* industry at the disaggregated levels *during* the whole growth process.

Ngai and Pissarides (2007)study structural change in a growth model with an arbitrary but finite number of sectors, which potentially allows for the life-cycle analysis of disaggregated industries. However, they do not treat capital accumulation as a major force to drive structural change, even in their appendix, where they introduce different capital intensities across different sectors. Nor do they attempt to keep track of the life cycles of each industry to explain their dynamics along the growth path. Moreover, structural change will be over in the long run since there are finite sectors in their model.

Our paper is also closely related to the strand of growth literature that studies the life cycle dynamics of industries, firms, establishments or products. The key mechanisms that drive the life cycle dynamics are different in different models. For example, some highlight the role of innovation and creative destruction (see Stokey, 1988; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Jovanovic and MacDonald, 1994); some highlight the role of specific intangible capital such as organizational capital (see Atkeson and Kehoe, 2005) or technology-specific or industry-specific human capital (see Chari and Hopenhayn, 1991; Rossi-Hansberg and Wright, 2007); some focus on productivity change and destruction shocks (see Hopenhayn, 1992; Luttmer, 2007; Samaniego, 2010), still others highlight demand shift due to consumers' heterogeneous preferences together with product awareness (Perla, 2013) or non-homothetic preferences (Matsuyama, 2002). Our paper differs from and complements these approaches by focusing on the role of endowment structure via the endogenous relative factor prices. For simplicity and clarity, we abstract away complications in other dimensions: there are no industry-specific or technology-specific factors, everything is deterministic, and all the technologies already exist and are freely available.

The paper is organized as follows: Section 2 documents and summarizes the empirical facts about industrial dynamics. Section 3 presents the static model. The dynamic model is analyzed in Section 4. Section 5 considers two extensions: an alternative dynamic setting and a more general CES aggregate production function. Section 6 concludes. Technical proofs and some other extensions are in the Appendix.[7]

[4] For instance, the Stone–Geary function is used in Laitner (2000), Kongsamut et al. (2001), Caselli and Coleman (2001), and Gollin et al. (2007). Hierarchic utility functions are adopted in Matsuyama (2002), Foellmi and Zweimuller (2008), Buera and Kaboski (2012a), among others.

[5] See, e.g., Ngai and Pissarides (2007), Hansen and Prescott (2002), Duarte and Restuccia (2010), Yang and Zhu (2013), Uy et al. (2013).

[6] Ngai and Samaniego (2011) study how R&D differs across industries and contributes to industry-specific TFP growth. Acemoglu (2007) argues that technology progress is endogenously biased toward utilizing the more abundant production factors, which indicates that endowment structure is also fundamentally important even in accounting for TFP growth itself.

[7] International trade could be another important force to drive structural change (see Ventura, 1997; Matsuyama, 2009; Uy et al., 2013). We show that our main results remain valid in a small open economy, both with and without international capital flows. It is relegated to the appendix due to space limits.

2. Empirical patterns of industrial dynamics

This section documents several important facts of industrial dynamics to motivate and guide our theoretical explorations. Ideally, we need a sufficiently long time series of production data for each industry at a sufficiently high-digit industry level. Due to limited availability of quality data, here we will focus on the manufacturing sector.

2.1. Evidence from US data

We use the NBER-CES Manufacturing Industry Data for the US, which adopts the 6-digit NAICS codes and covers 473 industries within the manufacturing sector from 1958 to 2005.[8]

It is found that there exists tremendous cross-industry heterogeneity in both physical capital intensities and productivities.[9] For example, among all industries in 1958, the highest capital–labor ratio is 522,510 US dollars per worker, which is 986 times larger than the lowest one; the highest capital expenditure share (one minus the ratio of labor expenditure to the value added) is 0.866 while the lowest share is 0.132. In 2005, the highest capital–labor ratio is still about 148 times higher than the lowest one. The highest labor productivity is 50.94 in the unit of thousand dollars/worker) in 1958, which is 35 times larger than that in the least-efficient industry. In 2005 the top-bottom cross-industry labor productivity difference is 92-fold (see Table A1 in the Appendix for detailed descriptive statistics).

In addition, the capital intensity (measured by the capital–labor ratio or the capital expenditure share) increases over time within each industry (see Table A1). This may be because each industry consists of many different products or techniques of different capital intensities and the production activity within each industry gradually switches from labor-intensive products/procedures/tasks to capital-intensive ones. Moreover, it is likely that the rank of capital intensity for two highly disaggregated industries can be reversed over time. These issues have long plagued empirical trade economists when testing the Heckscher–Ohlin trade theory because the standard classification of industries is not based on their capital intensities and also capital intensities of products in the same industry may change over time. Schott (2003) proposes to solve these problems by redefining the industries according to their capital–labor ratios. A similar practice is followed here. We first rank all the 22,275 observations (473 industries for 48 years, and 429 observations are dropped due to missing values in capital–labor ratio) by the capital–labor ratios in an increasing order, and then equally divide all these observations into 99 bins (newly-defined industries). Within each newly-defined industry, there are 225 observations. The first bin, called “industry 1”, is most labor-intensive by construction. Likewise, the most capital-intensive bin is “industry 99”. In this way, the rank of capital intensity among all these newly-defined industries is clear and time-invariant.[10] For the purpose of illustration, Fig. 1 plots the time series of the HP-filtered employment shares of six typical industries from 1958 to 2005.[11]

Except for a few anomalies, a discernible pattern emerges. The employment share decreases over time in the most labor-intensive industries, exhibits a hump shape in the “middle” capital-intensive industries, and increases over time in the most capital-intensive industries. If longer time series become available, one would expect to see the rising stage of the most labor-intensive industries before 1958 and perhaps the eventual decline of the capital-intensive industries after 2005. Similar patterns are also observed for the value-added share or when capital intensity is measured alternatively by the share of capital expenditure.[12] This non-monotonic development pattern of an industry is referred to as the *hump-shaped pattern* of industrial dynamics. Moreover, a more capital-intensive industry reaches its peak later, which is referred to as the *timing fact*. More formally, we regress the peak time of an industry's share (either employment share or value-added share) on its capital intensity. The results are in Table 1. It shows that the peak time of an industry is positively correlated with its capital intensity.

To further understand what determines the industrial structures and their dynamics, we run the following regression:

$$LS_{it} = \beta_0 + \beta_1 \left| \frac{K_{it}/L_{it} - K_t/L_t}{K_t/L_t} \right| + \beta_2 T_{it} + industry\_dummy + \varepsilon_{it}, \tag{1}$$

where LS_{it} is the ratio of newly defined industry i's employment to the total manufacturing employment at year t and $|K_{it}/L_{it} - K_t/L_t/K_t/L_t|$ is the absolute value of a normalized difference between industry i's capital–labor ratio and the aggregate capital–labor ratio at year t. T_{it} is the labor productivity of the newly-defined industry i at year t. The results are reported in Table 2. It shows that β_1 is negative and significant, indicating that an industry's employment share becomes smaller if the difference between the industry's capital intensity and the aggregate capital–labor ratio is larger, which suggests that if an industry's capital intensity and the economy's endowment structure are more congruent, that industry is

[8] The data set can be downloaded at http://www.nber.org/nberces.

[9] Note that the TFP growth rate obtained from the NBER-CES data set is industry-specific and hence the level of TFP is not directly comparable across industries. So we mainly report and use labor productivity for our cross-section regression analyses. However, the same pattern is still found robust when we use the TFP-measured productivity after normalizing the TFP values for all the industries in year 2002 to 100.

[10] A more detailed description for the method of industry reclassification is in the Appendix 8.

[11] The labor productivity, capital–labor ratio and capital expenditure share at in the newly defined “industry n” are calculated by using the aggregates of capital, labor, labor expenditure, and value added for all the observations at t in “industry n”. When generating Fig. 1, if there are missing values in the constructed time series for certain years, we replace the missing values by the simple average of the observations before and after. For how employment shares change over time in all the 99 industries within the manufacturing sector, refer to Figure A1 in the Appendix.

[12] Alternatively, when the observations are ranked according to the capital expenditure share, 75 observations with non-positive capital expenditure shares are dropped. So we end up with a sample of 22,200 observations, which are equally divided into 100 bins, with 222 observations in each bin.

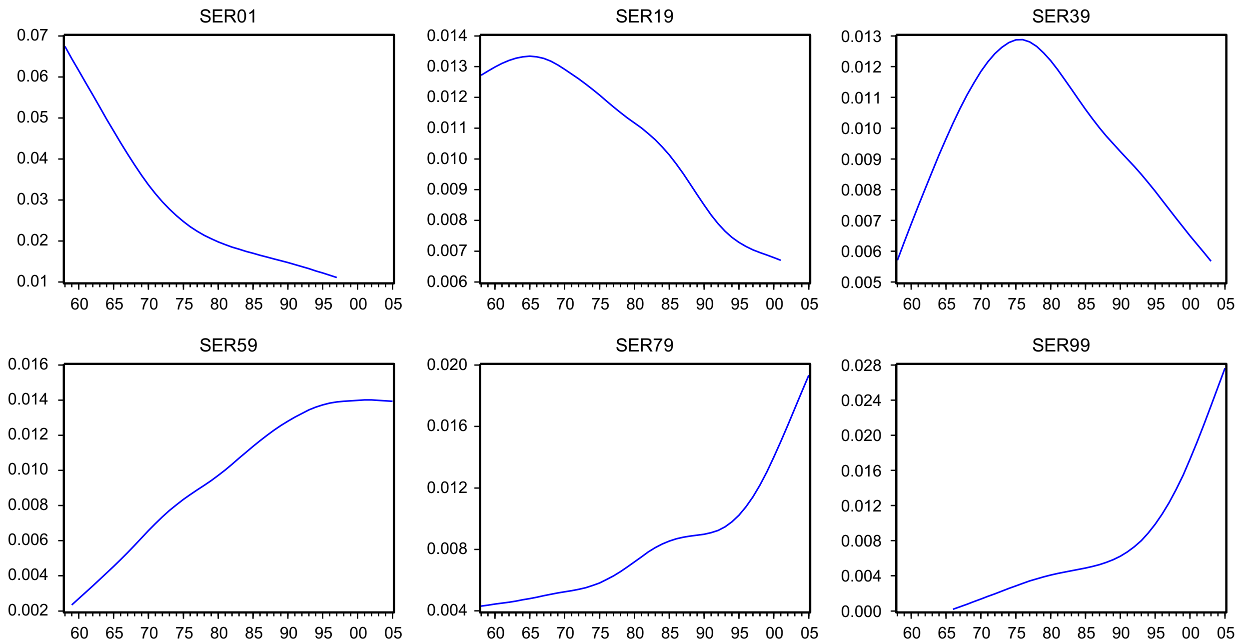

Fig. 1. HP-filtered employment shares of six typical “Industries” (Newly Defined) in US from 1958 to 2005. *Note*: The horizontal axis is the year (from 1958 to 2005), and the vertical axis is the employment share of “Industry 1 (19, 39, 59, 79 and 99)”. An industry with a higher index is more capitalintensive by construction. If there is a missing value in the constructed time series for a certain year, that missing value is replaced by the simple average of the observations immediately before and after that year. The HP filter parameter λ is set to 1000.

Table 1
Peak time of industries in US: 1958–2005.

Independent variable	Peak time of employment share		Peak time of value-added share	
	(1)	(2)	(3)	(4)
Capital–labor ratio	0.535***		0.553***	
	(0.070)		(0.070)	
Capital expenditure share		123.298***		97.863***
		(16.550)		(9.841)
Constant	1958.309***	1915.588***	1955.004***	1924.355***
	(3.371)	(8.876)	(3.625)	(5.525)
Observations	33	40	34	43
R-squared	0.651	0.594	0.661	0.707

Note: We drop industries monotonically declining (increasing) during entire time period, and a few industries which are not single peaked. Data source: NBER-CES manufacturing database.
*** Indicates significance at 1% level.

relatively larger. As shown in Table 2, this result is robust if capital intensity is alternatively measured by the capital expenditure share or if the employment share is replaced by the value-added share. This is called the *congruence fact*. In addition, an industry's employment share (or valued-added share) is significantly and positively correlated with the level of labor productivity.

How robust are these patterns, especially the hump-shaped pattern and the timing fact, to the alternative ways in which an industry is defined?[13] To address this concern, we return to the original definition of industries with 6-digit NAICS codes and run the following regression by including a quadratic term for time:

$$y_{it} = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 \cdot k_{it} \cdot t + \beta_4 T_{it} + \beta_5 D_i + \beta_6 GDPGR_t + \varepsilon_{it}, \tag{2}$$

where y_{it} is the output or value-added share of industry i in the total manufacturing sector at year t; k_{it} and T_{it} are the capital expenditure share and the labor productivity of industry i at year t, respectively; D_i is the industry dummy; $GDPGR_t$ is the

[13] As one may expect, a potential limitation of Schott's method in the dynamic context is that the newly-defined “industry 1” by construction has more data points for the earliest years and “industry 99” has disproportionately more data points for the latest years. However, it is not necessarily biased for most of the “middle industries.” Anyway, the robustness of all the patterns is worth checking using the traditional definition of industries, which we do below.

Table 2
Congruence fact of industry dynamics in US: 1958–2005.

Independent variable	Employment share × 1000		Value-added share × 1000	
	(1)	(2)	(3)	(4)
Coherence term 1	−3.035***		−3.603***	
	(0.266)		(0.262)	
Coherence term 2		−33.228***		−23.594***
		(1.562)		(1.509)
T	0.022***	0.018***	0.100***	0.134***
	(0.006)	(0.005)	(0.006)	(0.005)
Constant	14.196***	18.595***	14.191***	14.877***
	(0.991)	(1.030)	(0.975)	(0.995)
Industry dummies	Yes	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes	Yes
Observations	4515	4542	4515	4542
R-squared	0.133	0.201	0.296	0.381

Note: Coherence term 1 is the absolute value of a normalized difference between sector *i*'s capital-labor ratio and the aggregate capital-labor ratio in the manufacturing sectors at year *t*. Coherence term 2 is the absolute value of a normalized difference between sector *i*'s capital expenditure share and the average capital expenditure share in all manufacturing sectors at year *t*. Data source: NBER-CES manufacturing database.

*** Indicates significance at 1% level. In addition, the result of a Hausman test shows that the fixed effect model is statistically preferred to the random effect model.

Table 3
Hump-shaped pattern and timing fact of industrial dynamics in US: 1958–2005.

Independent variable	Full sample		Subsample	
	(1) Value-added share × 1000	(2) Output share × 1000	(3) Value-added share × 1000	(4) Output share × 1000
t	1.581***	1.197***	1.596***	1.205***
	(0.223)	(0.263)	(0.223)	(0.263)
t^2	−4.07e−04***	−3.08e−04***	−4.11e−04***	−3.10e−04***
	(5.61e−05)	(6.62e−05)	(5.63e−05)	(6.64e−05)
$t \times k$	0.001***	0.001***	0.001***	0.001***
	(7.80e−05)	(9.21e−05)	(7.82e−05)	(9.23e−05)
T	0.006***	0.004***	0.006***	0.004***
	(1.47e−04)	(1.73e−04)	(1.47e−04)	(1.74e−04)
GDPGR	−0.710*	−0.475	−0.688*	−0.463
	(0.415)	(0.489)	(0.416)	(0.491)
Constant	−1534***	−1163***	−1549***	−1171***
	(220.705)	(260.383)	(221.372)	(261.206)
Industry dummies	Yes	Yes	Yes	Yes
Observations	20,889	20,889	20,790	20,790
R-squared	0.862	0.884	0.862	0.884

Note: t, k, T and GDPGR represent Year, Capital expenditure share, Labor productivity and GDP growth rate, respectively. Standard errors are in parentheses. Column 3 and Column 4 report the results when we drop all the eleven industries that have data for only 12 years. Data source: NBER-CES manufacturing database. ** Denote significance at the 5 percent level, respectively.

* Denotes significance at the 10 percent level.

*** Denotes significance at the 1 percent level.

GDP growth rate; and ε_{it} is the error term. If the hump-shaped dynamic pattern is statistically valid, one should expect the coefficient for the quadratic term, β_2, to be negative and significant. In addition, after controlling for the labor productivity and the industry fixed effect, we know from (2) that industry i reaches its peak at $t_i^{\max} \equiv -(\beta_1 + \beta_3 \cdot k_{i,t})/2\beta_2$. That is, $\partial y_{it}/\partial t > 0$ if and only if $t < t_i^{\max}$. If the timing fact is statistically valid, we should expect $-\beta_3/\beta_2$ to be positive, or equivalently, β_3 should be positive when β_2 is negative. Moreover, the peak time $t_i^{\max}$ must be positive when β_1 is positive. Table 3 summarizes all the regression results, which confirm that these two patterns are indeed statistically significant, so the results are robust.

To check the robustness of the congruence fact, we use the original NAICS classification and run regression (1) again. The results are as follows (with standard errors in parentheses):

$$LS_{it} = \underset{(0.016)}{2.233} - \underset{(0.019)}{0.071}\left|\frac{K_{it}/L_{it} - K_t/L_t}{K_t/L_t}\right| - \underset{(1.04\times10^{-4})}{4.33 \times 10^{-4}} T_{it} + industry\_dummy.$$

Table 4
Hump-shaped pattern and timing fact of industrial dynamics cross countries: 1963–2009.

Independent variable	Full sample		Subsample	
	(1) Value-added share × 1000	(2) Output share × 1000	(3) Value-added share × 1000	(4) Output share × 1000
t	9.750**	9.432**	10.314**	12.267**
	(4.587)	(4.507)	(5.112)	(5.652)
t^2	−0.002**	−0.002**	−0.003**	−0.003**
	(0.001)	(0.001)	(0.001)	(0.001)
$t \times k$	4.01e−05***	2.54e−05*	2.42e−04***	1.51e−04**
	(1.37e−05)	(1.34e−05)	(6.85e−05)	(7.61e−05)
T	3.16e−09***	2.28e−10*	2.87e−09***	2.42e−10*
	(1.28e−10)	(1.25e−10)	(1.19e−10)	(1.32e−10)
GDPGR	−3.726	−1.585	0.831	4.515
	(3.874)	(3.811)	(5.247)	(5.809)
Constant	−9519**	−9206**	−10,050**	−12,009**
	(4556)	(4477)	(5077)	(5614)
Country × Industry dummies	Yes	Yes	Yes	Yes
Observations	47,328	47,179	27,236	27,447
R-squared	0.820	0.847	0.774	0.796

Note: t, k, T and GDPGR represent Year, Capital expenditure share of an industry, Labor productivity of an industry, and GDP growth rate for each specific country, respectively. Standard errors are in parentheses. Regressions (3) and (4) are run for the subsample which includes only those industries whose values are observed for over 40 years. Data source: UNIDO (1963–2009).

* Denotes significance at the 10 percent level.

** Denotes significance at the 5 percent level.

*** Denotes significance at the 1 percent level.

The coefficient β_1 is again negative and significant at the 99% significant level, confirming the congruence fact.[14] The employment share now is negatively correlated with the labor productivity.

Finally, we examine the correlation between the industrial capital–labor ratios and the labor productivities. The following regression results are obtained (with standard errors in parentheses):

$$T_{it} = 24.184 + \underset{(0.004)}{0.0415}(K_{it}/L_{it}). \tag{3}$$

It shows that industrial productivities and capital intensities are positively correlated at the 99% significant level. This positive correlation remains significant and robust when capital intensities are measured by capital expenditure shares and/or when controlling for industrial output (or value added, employment) and year dummies.

2.2. Evidence from cross-country data

We now turn to the evidence from the cross-country data, and we are particularly interested in the patterns for developing countries. Haraguchi and Rezonja (2010) explore the UNIDO data set of manufacturing development covering 135 countries from 1963 to 2006. Their data set adopts the 2-digit ISIC codes and entails 18 manufacturing sectors. They group all the observations together in their regressions. One of their most robust findings is that the logarithm of the value-added share of an industry in the total GDP can be best fitted by including a quadratic term of the logarithm of real GDP per capita in the regressions; the coefficient is both negative and significant after controlling for various country-specific factors (such as population and natural resource abundance) and country dummies. This is consistent with the pattern of the hump-shaped industrial dynamics documented earlier for the US data. Following Chenery and Taylor (1968) and Haraguchi and Rezonja (2010) group industries into three separate panels: "early sectors", "middle sectors" and "late sectors". They find that industries in "early sectors" reach their peaks at lower levels of GDP per capita ("earlier") than those in "middle sectors", which in turn reach their peaks "earlier" than those in "late sectors". In other words, a more capital-intensive industry reaches a peak at a higher level of GDP per capita ("later"), which is consistent with our timing fact. The congruence fact is also empirically found by Lin (2009) in the cross-country empirical analyses.

To further investigate the cross-country empirical evidence for industry dynamics, especially for developing countries, we extend regression (2) to a cross-country regression:

$$y_{itc} = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 \cdot k_{itc} \cdot t + \beta_4 T_{itc} + \beta_5 D_{ic} + \beta_6 GDPGR_{tc} + \varepsilon_{itc} \tag{4}$$

where subscript c represents a country index. The UNIDO manufacturing data set used in this paper covers 166 countries from 1963 to 2009 at the two-digit level (23 sectors). The results are summarized in Table 4.

[14] When the value-added share is used as the dependent variable, or when the capital expenditure share is used in the congruence term, the congruence fact cannot be found for the data with the NAICS classification.

Table 5
Hump-shaped pattern and timing fact of industrial dynamics in developing countries: 1963–2009.

Independent variable	India		A		B		C	
	Value-added share × 1000	Output share × 1000	Value-added share × 1000	Output share × 1000	Value-added share × 1000	Output share × 1000	Value-added share × 1000	Output share × 1000
t	144.706***	292.409***	104.543***	157.847**	13.926*	25.969***	14.016*	27.975***
	(42.073)	(98.763)	(28.761)	(61.281)	(8.302)	(6.958)	(8.498)	(7.227)
t^2	−0.036***	−0.074***	−0.026***	−0.040***	−0.004*	−0.007***	−0.004*	−0.007***
	(0.011)	(0.025)	(0.007)	(0.015)	(0.002)	(0.002)	(0.002)	(0.002)
$t \times k$	0.011**	0.003	0.013***	0.003	1.42e−04**	1.05e−04*	1.38e−04*	1.05e−04*
	(0.005)	(0.012)	(0.002)	(0.004)	(7.10e−05)	(6.07e−05)	(7.10e−05)	(6.16e−05)
T	0.001***	0.002***	4.74e−04***	7.15e−04***	1.81e−04***	8.93e−05***	1.73e−04***	8.93e−05***
	(5.96e−05)	(1.4e−04)	(2.66e−05)	(5.63e−05)	(6.60e−06)	(5.64e−06)	(6.73e−06)	(5.84e−06)
GDPGR	−19.639	−48.864	−22.213	−42.336	−3.200	1.424	−4.597	1.081
	(32.614)	(76.559)	(21.245)	(45.211)	(6.314)	(5.361)	(7.137)	(6.137)
Constant	−143,406***	−290,248***	−103,595***	−156,722**	−13,433	−25,474***	−13,527	−27,459***
	(41,945)	(98,461)	(28,669)	(61,085)	(8,246)	(6,910)	(8,440)	(7,177)
Country × industry dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	467	467	924	914	12,073	12,531	11,632	12,090
R-squared	0.920	0.762	0.903	0.746	0.836	0.900	0.824	0.885

Note: t, k, T and GDPGR represent Year, Capital expenditure share, Labor productivity and GDP growth rate, respectively. Standard errors are in parentheses. All the regressions are run for the various subsamples with industry values observed for more than 40 years. The first column is for India, one of the BRICS countries. BRICS is the acronym for the five largest emerging economies, i.e., Brazil, Russia, India, China and South Africa. Since the data set records no industry values for more than 40 years in Brazil, Russia and China, Column A is for only India and South Africa together. Column B is for all the developing countries (following the classification by the World Bank) excluding India and South Africa. Other developing countries include Philippines, Cyprus, Panama, Hungary, Kuwait, Costa Rica, Fiji, Sri Lanka, Turkey, Jordan, Pakistan, Tanzania, Malaysia, Singapore, Trinidad, Ecuador, Tunisia, Iran, Uruguay, Syria, Kenya, Malta, Malawi, Egypt, Columbia and Garner. Column C is for the subsample of developing countries whose average annual real GDP growth rates are more than 2%. Average annual real GDP growth rate is calculated by the authors based on the data from Penn World Table 8.0. Data Source: UNIDO (1963–2009).

* Denotes significance at the 10 percent level.
** Denotes significance at the 5 percent level.
*** Denotes significance at the 1 percent level.

The findings are consistent with the results in Table 3, which suggests that the industry dynamics patterns observed in the US are actually quite general and also true for other countries.

Next we check the robustness of the empirical patterns for the subsample that only consists of developing countries (following the World Bank's definition). The results are reported in Table 5.

As can be seen again, β_1 is positive, β_2 is negative and β_3 is positive. Moreover, these three coefficients are significant in almost all the cases. Therefore, it is concluded that the hump-shaped industry pattern and the timing fact are robust for developing countries as well.

2.3. *Summary of empirical facts*

Based on all the above empirical results, the four facts for industrial dynamics in the manufacturing sector can be summarized as follows.

Fact 1 (*cross-industry heterogeneity*): There exists tremendous cross-industry heterogeneity in capital–labor ratios, capital expenditure shares and labor productivities.

Fact 2 (*hump-shaped dynamics*): An industry typically exhibits a hump-shaped dynamic pattern: its value-added share first increases, reaches a peak, and then declines.

Fact 3 (*timing fact*): The more capital intensive an industry is, the later its value-added share reaches its peak.

Fact 4 (*congruence fact*): The further an industry's capital–labor ratio deviates from the economy's aggregate capital–labor ratio, the smaller is the industry's employment share.

In this paper, we take Fact 1 as exogenously given and develop a growth model with multiple industries of different capital intensities to simultaneously explain Facts 2, 3 and 4.

3. Static model

In this section, we characterize the equilibrium composition of industries and derive the endogenous aggregate production function for any given factor endowment.

3.1. Set-up

Consider an economy with a unit mass of identical households and infinite industries. Each household is endowed with L units of labor and E units of physical capital, which can be easily extended to incorporate intangible capital as well. A representative household consumes a composite final commodity C, which is produced by combining all the intermediate goods c_n, where $n \in \{0, 1, 2, \ldots\}$. Each intermediate good should be interpreted as an industry, although we will use "good" and "industry" interchangeably throughout the paper.

For simplicity, assume the production function of the final commodity is

$$C = \sum_{n=0}^{\infty} \lambda_n c_n, \tag{5}$$

where λ_n represents the marginal productivity of good n in the final good production.[15] We require $c_n \geq 0$ for any n. The final commodity serves as the numeraire. The utility function is *CRRA*:

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma}, \quad \text{where } \sigma \in (0, 1). \tag{6}$$

All the technologies exhibit constant returns to scale. In particular, good 0 is produced with labor only. One unit of labor produces one unit of good 0. To produce any good $n \geq 1$, both labor and capital are required and the production functions are Leontief[16]:

$$F_n(k, l) = \min\left\{\frac{k}{a_n}, l\right\}, \tag{7}$$

where a_n measures the capital intensity of good n. All the markets are perfectly competitive. Let p_n denote the price of good n. Let r denote the rental price of capital and w denote the wage rate. The zero profit condition for a firm implies that $p_0 = w$ and $p_n = w + a_n r$ for $n \geq 1$.

Without loss of generality, the industries are ordered such that a_n is increasing in n. Since the data suggest that a more capital-intensive technology is generally more productive (refer to regression result (3)), λ_n is assumed to be increasing in n. To obtain analytical solutions, we assume

$$\lambda_n = \lambda^n, \quad a_n = a^n, \tag{8}$$

$$a - 1 > \lambda > 1. \tag{9}$$

$a > \lambda$ must be imposed to rule out the trivial case that only the most capital-intensive good is produced in the static equilibrium, and the assumption is further strengthened to $a - 1 > \lambda$ to simplify the analysis as good 0 requires no capital.[17]

The household problem is to maximize (6) subject to the following budget constraint:

$$C = wL + rE. \tag{10}$$

3.2. Market equilibrium

We first establish that at most two goods are simultaneously produced in the equilibrium and that these two goods have to be adjacent in the capital intensities (see Appendix 1 for the proof). The intuition is the following. Suppose goods n and $n+1$ are produced for some $n \geq 1$, then the marginal rate of transformation (MRT) between the two intermediate goods must be equal to their price ratio: $MRT_{n+1,n} = \lambda = (p_{n+1})/p_n = (w + a^{n+1}r)/(w + a^n r)$, which yields

$$\frac{r}{w} = \frac{\lambda - 1}{a^n(a - \lambda)}. \tag{11}$$

Obviously, $MRT_{j,j+1} > p_j/(p_{j+1})$ whenever $r/w > (\lambda-1)/a^j(a-\lambda)$, and $MRT_{j,j-1} > p_j/(p_{j-1})$ whenever $r/w < (\lambda-1)/a^{j-1}(a-\lambda)$ for any $j = 1, 2, \ldots$ Therefore, when (11) holds, good $n+1$ must be strictly preferred to good $n+2$, because the MRT is larger than their price ratio. This means that $c_j = 0$ for all $j \geq n+2$. Using the same logic, we can also verify that $c_j = 0$ for all $1 \leq j \leq n-1$. In addition, condition (9) ensures that good 0 is not produced. Similarly, when goods 0 and 1 are produced in an equilibrium, good 1 is strictly preferred to any good $n \geq 2$.

[15] It is not unusual in the growth literature to assume perfect substitutability for the output across different production activities; see Hansen and Prescott (2002). We will relax this assumption and study the general CES function in Section 5.

[16] Leontief functions are also used in Luttmer (2007) and Buera and Kaboski (2012a,b). See Jones (2005) for more discussions of functional forms. It can be easily shown that our key qualitative results will remain valid when the production function is Cobb–Douglas, but that will enormously increase the nonlinearity of the problem in the multiple-sector environment, making it much harder to obtain closed-form solutions, especially for the dynamic analysis.

[17] If $\lambda = 1$, the equilibrium would be trivial because only good 0 is produced in this linear case. Later, we will allow for $\lambda = 1$ when (5) is replaced by a general CES function in Section 5.

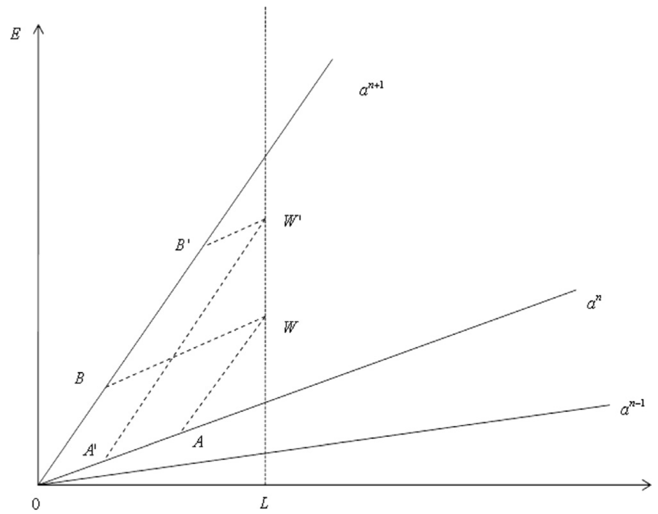

Fig. 2. How endowment structure determines equilibrium industries.

Table 6
Static equilibrium.

$0 \le E < aL$	$a^n L \le E \le a^{n+1} L$ for $n \ge 1$
$c_0 = L - \frac{E}{a}$	$c_n = \frac{La^{n+1} - E}{a^{n+1} - a^n}$
$c_j = 0$ for $\forall j \ne 0, 1$	$c_j = 0$ for $\forall j \ne n, n+1$
$\dfrac{r}{w} = \dfrac{\lambda - 1}{a}$	$\dfrac{r}{w} = \dfrac{\lambda - 1}{a^n(a-\lambda)}$
$C = L + (\lambda - 1)\frac{E}{a}$	$C = \frac{\lambda^{n+1} - \lambda^n}{a^{a+1} - a^n}E + \frac{\lambda^n(a-\lambda)}{a-1}L$
$\Leftrightarrow E_{0,1} = \frac{a}{\lambda - 1}(C - L)$	$\Leftrightarrow E_{n,n+1} = \left[C - \frac{\lambda^n(a-\lambda)}{a-1}L\right]\frac{a^{a+n} - a^n}{\lambda^{n+1} - \lambda^n}.$

Note: The static equilibrium is summarized verbally in the following proposition.

The market clearing conditions for labor and capital are given respectively by

$$c_n + c_{n+1} = L, \tag{12}$$

$$c_n a^n + c_{n+1} a^{n+1} = E. \tag{13}$$

The market equilibrium can be illustrated in Fig. 2, where the horizontal and vertical axes are labor and capital, respectively. Point O is the origin and Point $W = (L, E)$ denotes the endowment of the economy. When $a^n L < E < a^{n+1} L$, as shown in the current case, only goods n and $n+1$ are produced. The factor market clearing conditions, (12) and (13), determine the equilibrium allocation of labor and capital in industries n and $n+1$, which are represented respectively by vector OA and vector OB in the parallelogram $OAWB$. $\overrightarrow{Oa^n} = (1, a^n)c_n$ and $\overrightarrow{Oa^{n+1}} = (1, a^{n+1})c_{n+1}$ are the vectors of factors used in producing c_n and c_{n+1} in the equilibrium. If the capital increases so the endowment point moves from W to W', the new equilibrium becomes parallelogram $OA'W'B'$ so that c_n decreases but c_{n+1} increases. When $E = a^n L$, only good n is produced. Similarly, if $E = a^{n+1} L$, only good $n+1$ is produced.[18]

More precisely, the equilibrium output of each good c_n, the relative factor prices r/w, and the corresponding aggregate output C are summarized in Table 6.

Proposition 1. *Generically, there exist only two industries whose capital intensities are the most adjacent to the aggregate capital–labor ratio, E/L. As E/L increases, each industry n ($n \ge 1$) exhibits a hump shape: the output first remains zero, then increases and reaches its peak and then declines, and finally returns to zero and is fully replaced by the industry with the next higher capital intensity.*

The equilibrium outcome, as summarized in the above proposition and Table 6, is logically consistent with Facts 2 through 4 documented in Section 2. Table 6 also shows that the aggregate production function (C as a function of L and E) has different forms when the endowment structures are different, reflecting the endogenous structural change in the underlying industries. Accordingly, the coefficient right before E in the endogenous aggregate production function is the

[18] This graph may appear similar to the Lerner diagram in the H–O trade models with multiple diversification cones (see Leamer, 1987). However, the mechanism in our autarky model is different from the international specialization mechanism in the trade literature.

rental price of capital, and the coefficient before L is the wage rate. So the relative factor price is $r/w = (\lambda-1)/a^n(a-\lambda)$ when $E \in [a^nL, a^{n+1}L)$, and it declines in a stair-shaped fashion as E increases. This discontinuity results from the Leontief assumption. Observe that the capital income share in the total output is given by

$$\frac{rE}{rE+wL} = \frac{\left(\frac{\lambda-1}{a-1}\right)E}{\frac{\lambda-1}{a-1}E + \frac{a^n(a-\lambda)}{a-1}L} \tag{14}$$

when $E \in [a^nL, a^{n+1}L)$ for any $n \geq 1$. So the capital income share monotonically increases with capital within each diversification cone and then suddenly drops to $(\lambda-1)/(a-1)$ as the economy enters a different diversification cone, but the capital income share always stays within the interval $[(\lambda-1)/(a-1), (\lambda-1)a/(a-1)\lambda]$ for any $n \geq 1$. This is consistent with the Kaldor fact that the capital income share is fairly stable over time.[19]

Table 6 also shows that the equilibrium industrial output and the industrial employment are not affected by λ. The intuition is straightforward: an increase in λ raises the relative productivity of good $n+1$ versus good n. But their relative price $(p_{n+1})/p_n$ is also equal to λ, thus these two forces exactly cancel out each other. However, this result is no longer valid when the substitution elasticity between different goods is less than infinity, which we will show in Section 5.

4. Dynamic model

In this section, a dynamic model will be developed to fully characterize the industrial dynamics along the growth path of the aggregate economy, where the capital changes endogenously over time. Empirical evidence will be also discussed.

4.1. Theoretical model

We will first describe the model environment and characterize the dynamic equilibrium, and then discuss how this model deviates from the standard setting.

4.1.1. Environment

There are two sectors in the economy: a sector producing capital goods and a sector producing consumption goods. Capital goods and consumption goods are distinct in nature and not substitutable. Moreover, they are produced with different technologies. Capital goods are produced using an AK technology: One unit of capital good produces A units of new capital goods, where A captures the effect of learning by doing. It also highlights the feature that the technology progress is investment-specific, so it occurs in the capital (investment) goods sector rather than in the consumption goods sector (see Greenwood et al., 1997).[20] Let $K(t)$ denote the capital *stock* available at the beginning of time t, so the output *flow* coming out of the capital good sector is $AK(t)$, which is then split between two different usages:

$$AK(t) = X(t) + E(t), \tag{15}$$

where $X(t)$ denotes capital investment and $E(t)$ denotes the *flow* of capital used to produce consumption goods at t. $E(t)$ fully depreciates, so capital in the whole economy accumulates as follows:

$$\dot{K}(t) = X(t) - \delta K(t), \tag{16}$$

where δ is the depreciation rate in the capital goods sector. Substituting (15) into the above equation and defining $\xi = A-\delta$, we obtain

$$\dot{K}(t) = \xi K(t) - E(t).$$

At time t, capital $E(t)$ and labor L (assumed to be constant) produce all the intermediate goods $\{c_n(t)\}_{n=0}^{\infty}$ with technologies specified by (7), which are ultimately combined to produce the final consumption good $C(t)$ according to (5). Based on Table 6, define

$$F(E,L) \equiv \begin{cases} \frac{(\lambda-1)}{a}E + L & \text{if } 0 \leq E < aL \\ \frac{\lambda^{n+1}-\lambda^n}{a^{n+1}-a^n}E + \frac{\lambda^n(a-\lambda)}{a-1}L & \text{if } a^nL \leq E < a^{n+1}L \text{ for } n \geq 1 \end{cases}, \tag{17}$$

which is the endogenous aggregate production function derived in Section 3. Therefore,

$$C(t) = F(E(t), L) = r(t)E(t) + w(t)L, \tag{18}$$

[19] See Barro and Sala-i-Martin (2003) for more discussion on the robustness of the Kaldor facts.

[20] Notice that this dynamic setting differs from the most standard setting. For the sake of expositional convenience, detailed comparisons and justifications are postponed to Sections 4.1.3 and 5.

where $r(t)$ and $w(t)$ are the rental price for capital and the wage rate at time t, respectively. With some abuse of notation, let $E(C(t))$ denote the total amount of capital goods needed to produce final consumption goods $C(t)$, so $F(E(C(t), L) \equiv C(t)$. Final consumption goods C and all the intermediate goods $\{c_n\}_{n=0}^{\infty}$ are non-storable.

By the second welfare theorem, one can characterize the competitive equilibrium by resorting to the following social planner problem:

$$\max_{C(t)} \int_0^{\infty} \frac{C(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt \tag{19}$$

subject to

$$\begin{aligned} &\dot{K}(t) = \xi K(t) - E(C(t)), \\ &K(0) = K_0 \text{ is given,} \end{aligned} \tag{20}$$

where ρ is the time discount rate. Following conventions, we assume $\xi - \rho > 0$ to ensure positive consumption growth and, to exclude the explosive solution, we also assume $(\xi-\rho)/\sigma(1-\sigma) < \rho$. Putting them together, we impose

$$0 < \xi - \rho < \sigma\xi. \tag{21}$$

The social planner decides the intertemporal consumption flow $C(t)$ and makes optimal investment decisions $X(t)$, which in turn determine the evolution of the endowment structure $K(t)/L$ and the optimal amount of capital allocated for consumption goods production $E(t)$. Note that, at any given time t, once $E(t)$ is determined, the optimization problem for the whole consumption goods sector is exactly the same as the static problem in Section 3. The bottom row of Table 6 implies that $E(C)$ is a strictly increasing, continuous, piece-wise linear function of C. It is not differentiable at $C = \lambda^i L$, for any $i = 0, 1, \ldots$ Therefore, the above dynamic problem may involve changes in the functional form of the state equation: Eq. (20) can be explicitly rewritten as

$$\dot{K} = \begin{cases} \xi K & \text{when } C < L \\ \xi K - E_{0,1}(C) & \text{when } L \le C < \lambda L \\ \xi K - E_{n,n+1}(C) & \text{when } \lambda^n L \le C < \lambda^{n+1} L \quad \text{for } n \ge 1 \end{cases},$$

where $E_{n,n+1}(C)$ is defined in the bottom row of Table 6 for any $n \ge 0$.

4.1.2. Equilibrium characterization

It is straightforward to verify that the objective function is strictly increasing, differentiable and strictly concave while the constraint set forms a continuous convex-valued correspondence, hence the equilibrium must exist and also be unique.

Let t_0 denote the *last* time point when aggregate consumption equals L (that is, only good 0 is produced), and t_n denote the *first* time point when $C = \lambda^n L$ (that is, only good n is produced) for $n \ge 1$. As can be verified later, aggregate consumption C is monotonically increasing over time in equilibrium, hence the problem can also be written as

$$\max_{C(t)} \int_0^{t_0} \frac{C(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt + \sum_{n=0}^{\infty} \int_{t_n}^{t_{n+1}} \frac{C(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t}\, dt$$

subject to

$$\dot{K} = \begin{cases} \xi K & \text{when } 0 \le t \le t_0 \\ \xi K - E_{0,1}(C) & \text{when } t_0 \le t \le t_1 \\ \xi K - E_{n,n+1}(C) & \text{when } t_n \le t \le t_{n+1} \quad \text{for } n \ge 1 \end{cases},$$

$$K(0) = K_0 \text{ is given,}$$

where t_n is to be endogenously determined for any $n \ge 0$.

Table 6 indicates that goods 0 and 1 are produced during the time period $[t_0, t_1]$ and $E(C) = E_{0,1}(C) \equiv (a/(\lambda-1))(C-L)$. When $t_n \le t \le t_{n+1}$ for $n \ge 1$, goods n and $n+1$ are produced. Correspondingly, $E(C) = E_{n,n+1}(C) \equiv [C-(\lambda^n(a-\lambda)/(a-1))L](a^{n+1}-a^n)/(\lambda^{n+1}-\lambda^n)$. If K_0 is sufficiently small (this is more precisely shown below), then there exists a time period $[0, t_0]$ in which only good 0 is produced and all the working capital is saved for the future, so $E=0$ when $0 \le t \le t_0$. If K_0 is large, on the other hand, the economy may start by producing goods h and $h+1$ for some $h \ge 1$, so $t_0 = t_1 = \cdots = t_h = 0$ in equilibrium.

To solve the above dynamic problem, following Kamien and Schwartz (1991), we set the *discounted-value* Hamiltonian in the interval $t_n \le t \le t_{n+1}$, and use subscripts "$n, n+1$" to denote all the variables during this time interval:

$$H_{n,n+1} = \frac{C(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t} + \eta_{n,n+1}\left[\xi K(t) - E_{n,n+1}(C(t))\right] + \zeta_{n,n+1}^{n+1}\left(\lambda^{n+1}L - C(t)\right) + \zeta_{n,n+1}^{n}\left(C(t) - \lambda^n L\right) \tag{22}$$

where $\eta_{n,n+1}$ is the co-state variable, $\zeta_{n,n+1}^{n+1}$ and $\zeta_{n,n+1}^{n}$ are the Lagrangian multipliers for the two constraints $\lambda^{n+1}L - C(t) \ge 0$

and $C(t)-\lambda^n L \geq 0$, respectively. The first-order condition and Kuhn–Tucker conditions are

$$\frac{\partial H_{n,n+1}}{\partial C} = C(t)^{-\sigma} e^{-\rho t} - \eta_{n,n+1} \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} - \zeta^{n+1}_{n,n+1} + \zeta^n_{n,n+1} = 0,$$
$$\zeta^{n+1}_{n,n+1}(\lambda^{n+1}L - C(t)) = 0, \quad \zeta^{n+1}_{n,n+1} \geq 0, \ \lambda^{n+1}L - C(t) > 0,$$
$$\zeta^n_{n,n+1}(C(t)-\lambda^n L) = 0, \quad \zeta^n_{n,n+1} \geq 0, \ C(t)-\lambda^n L \geq 0. \tag{23}$$

Moreover,

$$\eta'_{n,n+1}(t) = -\frac{\partial H_{n,n+1}}{\partial K} = -\eta_{n,n+1}\xi. \tag{24}$$

In particular, when $C(t) \in (\lambda^n L, \lambda^{n+1} L)$, $\zeta^{n+1}_{n,n+1} = \zeta^n_{n,n+1} = 0$, and Eq. (23) becomes

$$C(t)^{-\sigma} e^{-\rho t} = \eta_{n,n+1} \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}. \tag{25}$$

The left-hand side is the marginal utility gain from increasing one unit of aggregate consumption, while the right-hand side is the marginal utility loss due to the decrease in capital because of that additional unit of consumption, which by the chain's rule can be decomposed into two multiplicative terms: the marginal utility of capital $\eta_{n,n+1}$ and the marginal capital requirement for each additional unit of aggregate consumption $(a^{n+1}-a^n)/(\lambda^{n+1}-\lambda^n)$ (see Table 6). Taking the logarithm on both sides of Eq. (25) and differentiating with respect to t, one can obtain the consumption growth rate from the regular Euler equation:

$$g_c \equiv \frac{\dot{C}(t)}{C(t)} = \frac{\xi-\rho}{\sigma}, \tag{26}$$

for $t_n \leq t \leq t_{n+1}$ for any $n \geq 0$. The strictly concave utility function implies that the optimal consumption flow $C(t)$ must be continuous and sufficiently smooth (without kinks); hence (26) implies

$$C(t) = C(t_0) e^{g_c(t-t_0)} \quad \text{for any } t \geq t_0 > 0. \tag{27}$$

According to Kamien and Schwartz (1991), there are two additional necessary conditions at $t = t_{n+1}$:

$$H_{n,n+1}(t_{n+1}) = H_{n+1,n+2}(t_{n+1}), \tag{28}$$

$$\eta_{n,n+1}(t_{n+1}) = \eta_{n+1,n+2}(t_{n+1}). \tag{29}$$

Substituting Eqs. (28) and (29) into (22), one can verify that $K^-(t_{n+1}) = K^+(t_{n+1})$. In other words, $K(t)$ is also continuous. Observe that

$$C(t_0) e^{g_c(t_n - t_0)} = C(t_n) = \lambda^n L \quad \text{when } t_0 > 0, \tag{30}$$

which implies

$$t_n = \frac{\log \frac{\lambda^n L}{C(t_0)} + \frac{\xi-\rho}{\sigma} t_0}{g_c} \quad \text{when } t_0 > 0. \tag{31}$$

Define $m_n \equiv t_{n+1} - t_n$, which measures the length of the time period during which both good n and good $n+1$ are produced (that is, the duration of the diversification cone for good n and good $n+1$). Obviously,

$$m_n = m \equiv \frac{\log \lambda}{g_c}. \tag{32}$$

The comparative statics for Eq. (32) is summarized in the following proposition.

Proposition 2. *The full life span of each industry $n \geq 1$ is equal to $2m$. The speed of industrial upgrading (measured by frequency $1/2m$) decreases with the productivity parameter λ but increases with the aggregate growth rate g_c. More precisely, the industrial upgrading is faster when technological efficiency ξ increases, or the intertemporal elasticity of substitution $1/\sigma$ increases, or the time discount rate ρ decreases.*

The intuition for the proposition is the following. Suppose good n and good $n+1$ are produced. When the productivity parameter λ is larger, the marginal productivity of good n (λ^n) becomes bigger, making it pay to stay at good n longer; but the marginal productivity of good $n+1$ (λ^{n+1}) also becomes bigger, making it optimal to leave good n and move to good $n+1$ more quickly. It turns out that the first effect dominates the second effect because (9) implies that, by climbing up the industrial ladder, the productivity gain λ is sufficiently small relative to the additional capital cost reflected by the cost parameter a. Thus the net effect is that industrial upgrading slows down. On the other hand, industrial upgrading is faster when the consumption growth rate g_c increases because larger consumption is supported by more capital-intensive industries, as implied by Table 6.

When the household is more impatient (larger ρ), it will consume more and save less and hence capital accumulation becomes slower and thus the endowment-driven industrial upgrading also becomes slower. When the production of the capital good becomes more efficient (ξ), capital can be accumulated faster, so the upgrading speed is increased. When the aggregate consumption is more substitutable across time (larger $1/\sigma$), the household is more willing to substitute current consumption for future consumption, which also boosts saving and then causes quicker industrial upgrading.

We are now ready to derive the industrial dynamics for the entire period. The industrial dynamics depend on the initial capital stock, $K(0)$. It is shown in the Appendix that there exists a series of increasing constants, $\vartheta_0, \vartheta_1, \ldots, \vartheta_n, \vartheta_{n+1}, \ldots$, such that if $0 < K(0) \le \vartheta_0$, the economy will start by producing good 0 only until the capital stock reaches ϑ_0 (Appendix 3 fully characterizes this case); if $\vartheta_n < K(0) \le \vartheta_{n+1}$, the economy will start by producing goods n and $n+1$ for any $n \ge 0$. Furthermore, it can be shown that $K(t_n) \equiv \vartheta_n$ for any $K(0) < \vartheta_n$. That is, irrespective of the level of initial capital stock, the economy always starts to produce good $n+1$ whenever its capital stock reaches ϑ_n.

To be more concrete, consider the case when $\vartheta_0 < K(0) \le \vartheta_1$, where the threshold values ϑ_0 and ϑ_1 can be explicitly solved (see Appendix 2). That is, the economy will start by producing goods 0 and 1. Using Eq. (27) and Table 6, we know that when $t \in [0, t_1]$,

$$E(t) = \frac{a}{\lambda-1}(C(t)-L) = \frac{a}{\lambda-1}\left(C(0)e^{((\xi-\rho)/\sigma)t} - L\right).$$

Correspondingly,

$$\dot{K} = \xi K(t) - \frac{a}{\lambda-1}\left(C(0)e^{((\xi-\rho)/\sigma)t} - L\right).$$

Solving this first-order differential equation with the condition $K(0) = K_0$, one obtains

$$K(t) = \frac{-\dfrac{aC(0)}{\lambda-1}}{\dfrac{\xi-\rho}{\sigma}-\xi}e^{((\xi-\rho)/\sigma)t} + \frac{-aL}{\xi(\lambda-1)} + \left[K_0 + \frac{\dfrac{aC(0)}{\lambda-1}}{\dfrac{\xi-\rho}{\sigma}-\xi} + \frac{aL}{\xi(\lambda-1)}\right]e^{\xi t},$$

which yields

$$\vartheta_1 \equiv K(t_1) = \frac{-\dfrac{a\lambda L}{\lambda-1}}{\dfrac{\xi-\rho}{\sigma}-\xi} + \frac{-aL}{\xi(\lambda-1)} + \left[K_0 + \frac{\dfrac{aC(0)}{\lambda-1}}{\dfrac{\xi-\rho}{\sigma}-\xi} + \frac{aL}{\xi(\lambda-1)}\right]\left(\frac{\lambda L}{C(0)}\right)^{\xi\sigma/(\xi-\rho)}.$$

When $t \in [t_n, t_{n+1}]$, for any $n \ge 1$, the transition equation of capital stock (20) becomes

$$\dot{K} = \xi K(t) - \left[C(0)e^{(\xi-\rho)/\sigma t} - \frac{\lambda^n(a-\lambda)}{a-1}L\right]\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} \quad \text{when } t \in [t_n, t_{n+1}] \quad \text{for any } n \ge 1,$$

which yields

$$K(t) = \alpha_n + \beta_n e^{((\xi-\rho)/\sigma)t} + \gamma_n e^{\xi t} \quad \text{when } t \in [t_n, t_{n+1}] \quad \text{for any } n \ge 1 \tag{33}$$

where

$$\alpha_n = -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right)\frac{\lambda^n(a-\lambda)L}{\xi(a-1)},$$

$$\beta_n = -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right)\frac{C(0)}{\left(\dfrac{\xi-\rho}{\sigma}-\xi\right)},$$

$$\gamma_n = \left[\frac{\lambda^n L}{C(0)}\right]^{-\xi\sigma/(\xi-\rho)}\left\{\vartheta_n + \frac{(a^{n+1}-a^n)L}{\lambda-1}\left[\frac{1}{\left(\dfrac{\xi-\rho}{\sigma}-\xi\right)} + \frac{(a-\lambda)}{\xi(a-1)}\right]\right\}.$$

Again the endogenous change in the functional form of the capital accumulation path (33) reflects the structural changes that underlie the aggregate economic growth. Note that $\{\vartheta_n\}_{n=2}^{\infty}$ are all constants, which can be sequentially pinned down: $\vartheta_n \equiv K(t_n)$ can be computed from Eq. (33) with $K(t_{n-1})$ known.

For each individual industry, Eq. (27) and Table 6 jointly imply

$$c_n^*(t) = \begin{cases} \dfrac{C(0)e^{((\xi-\rho)/\sigma)t}}{\lambda^n-\lambda^{n-1}} - \dfrac{L}{\lambda-1} & \text{when } t \in [t_{n-1}, t_n] \\ -\dfrac{C(0)e^{((\xi-\rho)/\sigma)t}}{\lambda^{n+1}-\lambda^n} + \dfrac{\lambda L}{\lambda-1} & \text{when } t \in [t_n, t_{n+1}] \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } n \ge 2$$

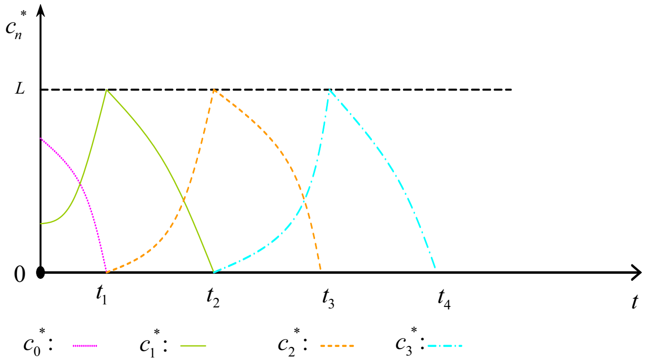

Fig. 3. How industries evolve over time when $K_0 \in (\vartheta_0, \vartheta_1)$. *Note*: the horizontal axis is time and the vertical axis is the consumption of good n. The purple line represents good 0; the green line represents good 1, the yellow line represents good 2, and the blue line represents good 3. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

$$c_1^*(t) = \begin{cases} \dfrac{C(0)e^{((\xi-\rho)/\sigma)t} - L}{\lambda - 1} & \text{when } t \in [0, t_1] \\ -\dfrac{C(0)e^{((\xi-\rho)/\sigma)t}}{\lambda^2 - \lambda} + \dfrac{\lambda L}{\lambda - 1} & \text{when } t \in [t_1, t_2] \\ 0 & \text{otherwise} \end{cases},$$

$$c_0^*(t) = \begin{cases} L - \dfrac{C(0)e^{((\xi-\rho)/\sigma)t} - L}{\lambda - 1} & \text{when } t \in [0, t_1] \\ 0 & \text{otherwise} \end{cases},$$

where $C(0)$ can be uniquely determined by the transversality condition (see Appendix 2) and the endogenous time points t_n are given by (31) for any $n \geq 1$. Recall $t_0 = 0$ in this case. The above mathematical equations fully characterize the industrial dynamics for each industry over the whole life cycle while aggregate consumption growth is still given by (26). If the initial capital stock is sufficiently small such that $K_0 < \vartheta_0$ as characterized in Appendix 3, then the economy will first have a constant output level equal to L (Malthusian regime) until the capital stock $K(t) = \vartheta_0$, which occurs at $t_0 > 0$, after which the aggregate consumption growth rate permanently changes to $(\xi - \rho)/\sigma$ (Solow regime). All these mathematical results can be read as follows:

Proposition 3. *There exists a unique and strictly increasing sequence of endogenous threshold values for capital stock, $\{\vartheta_i\}_{i=0}^{\infty}$, which are independent of the initial capital stock $K(0)$. The economy starts to produce good n when its capital stock $K(t)$ reaches ϑ_{n-1} for any $n \geq 1$. $K(t)$ evolves following Eq. (33), while total consumption $C(t)$ remains constant at L until t_0, after which it grows exponentially at the constant rate $(\xi - \rho)/\sigma$. The output of each industry follows a hump-shaped pattern: When capital stock $K(t)$ reaches ϑ_{n-1}, industry n enters the market and booms until capital stock $K(t)$ reaches ϑ_n; its output then declines and finally exits from the market at the time when $K(t)$ reaches ϑ_{n+1}.*

The industrial dynamics characterized in Proposition 3 are depicted in Fig. 3.

It is clear that our theoretical predictions for the industrial dynamics are well consistent with Facts 2–4 established in Section 2. Meanwhile, the theoretical results are also consistent with the Kaldor facts that the growth rate of total consumption remains constant and the capital income share is relatively stable, as shown in Eq. (14).

4.1.3. Discussion

Now we briefly discuss how this dynamic setting deviates from the most standard environment (standard setting, henceforth) and why the current alternative setting is desirable.[21] The standard setting postulates that there is one all-purpose final good, which can be either consumed or used for investment (think about coconut), so the production of consumption goods and capital goods are not separated. Thus (16) is still valid, but (15) and (18) are now replaced by

$$C(t) + X(t) = F(K(t), L) = r(t)K(t) + w(t)L, \tag{34}$$

where function $F(K(t), L)$ is defined in (17) with E being replaced by K. Combining (34) and (16) yields

$$\dot{K}(t) = F(K(t), L) - C(t) - \delta K(t), \tag{35}$$

which differs from (20). The social planner then solves (19) subject to (35) and that $K(0) = K_0$ is given. In contrast, our multi-sector model assumes that capital goods and consumption goods are different goods, which are used for different purposes

[21] Discussions in this subsection and in Section 5.1 are inspired by the referee.

and also produced with different technologies. In particular, technological progress takes place in the capital goods sector in an AK fashion, but there is no technological progress in the consumption goods sector.

There are four main reasons that we prefer the current setting over the standard setting.

First, generally speaking, due to their technological nature, capital goods (such as machines) usually cannot be directly used for consumption, nor can consumption goods (such as shirts) be used for capital investment, so the assumptions in the current setting are more realistic. Even for purely theoretical purposes, such a dichotomy has a long tradition in the growth literature, dating at least back to Uzawa (1961).

Second, the current setting is more consistent with two important empirical facts than the standard setting. Hsieh and Klenow (2007) show that the price of capital goods relative to consumption goods is systematically higher in poor countries, but the standard setting implies that this relative price is always equal to one, independent of the income level. However, this fact can be explained in the current setting. To see this, note that (17) implies the following relative price of capital goods to consumption goods:

$$r \equiv \begin{cases} \dfrac{\lambda-1}{a} & \text{if } 0 \le E < aL \\ \dfrac{\lambda^{n+1}-\lambda^{n}}{a^{n+1}-a^{n}} & \text{if } a^{n}L \le E < a^{n+1}L \text{ for } n \ge 1 \end{cases}.$$

Since E/L is larger in richer countries, n is larger, and therefore r is smaller because r decreases with n due to (9). Another fact which contradicts the predictions of the standard setting is that this relative price keeps falling dramatically over time within the same country such as the US, which is well established in the literature on investment-specific technological progress (see Greenwood et al., 1997; Cummins and Violante, 2002). In contrast, this fact is consistent with our setting, because industrial upgrading is perpetual (n keeps increasing) when the aggregate economy attains sustainable growth ($g_c > 0$), so the relative price $r(t)$ keeps falling over time. In fact, Proposition 2 further states that more efficient production in capital goods (higher ξ) leads to faster industrial upgrading (n increases faster as life spans of industries become shorter), thus $r(t)$ decreases faster.

Third, the primary purpose of our model is to highlight the role of capital accumulation in driving industrial dynamics, so it is conceptually clearer to explicitly distinguish capital goods from non-capital goods. Also, motivated by the literature on investment-specific technological progress, we assume the capital goods sector makes faster technological progress than the consumption goods sector.

Fourth, it turns out that closed-form solutions are obtained both for the life-cycle dynamics of each industry and for the aggregate growth path in our current setting, but we lose this high tractability in the standard setting. To make this point more clearly, in Section 5.1, we further explore the implications of the standard setting (with technological progress in the consumption goods sector).

4.2. Empirical evidence

Propositions 2 and 3 predict that the industrial dynamics exhibit a hump-shaped pattern and the life span of an industry ($2m$) decreases with the aggregate growth rate (g_c) but increases with its productivity level (recall that the productivity level is strictly increasing in λ). To empirically test these predictions, we first compute the life span of industry i, d_i, based on regression (2) and Table 3 in Section 2. Note that as the right-hand side of equation (2) is a quadratic function of time t, there are two roots of the parabola t_i^1 and t_i^2, at which the output (or value-added) share of industry i, y_{it}, equals zero. Therefore, the whole life span of industry i, can be measured by the distance of the two roots of the parabola:

$$d_{it} = \frac{\sqrt{(\beta_1+\beta_3 k_{it})^2 - 4\beta_2(\beta_0+\beta_4 T_{it}+\beta_5 D_i+\beta_6 GDPGR_t)}}{-\beta_2}.$$

From Table 3, we see that $\beta_2 < 0$, so we immediately have that $sign(\partial d_i/\partial T_{i,t}) = \text{sign}(\beta_4) > 0$ and $\text{sign}(\partial d_i/\partial GDPGR_t) = \text{sign}(\beta_6) < 0$. This empirical evidence supports the theoretical predictions that an industry's life span increases with the industrial labor productivity ($\partial d_i/\partial T_{i,t} > 0$) but decreases with the aggregate GDP growth rate ($\partial d_i/\partial GDPGR_t < 0$). For developing countries, note that, based on Tables 4 and 5, the effect of labor productivity on the life span of an industry is still consistent with the model prediction. However, the effect of aggregate growth rate (β_6) is not significant, although the sign is still consistent with the model prediction in most of the subsample regression results.[22]

5. Extensions

The purpose of this section is to show that the key results remain robust to alternative or more general settings.

[22] This may suggest that further research is needed to better understand the interaction between aggregate growth and industry dynamics for developing countries.

5.1. Alternative dynamic setting

In this subsection, we explore industry dynamics and aggregate growth in an alternative and more standard environment, where all capital is used to produce one final good, which can be either consumed or saved as capital investment.

Suppose the aggregate production function becomes

$$A(t)F(K(t),L), \tag{36}$$

where function $F(K(t),L)$ is defined as in (36) with $E(t)$ being replaced by $K(t)$, and $A(t)$ captures the exogenous Hicks-neutral technology progress.[23] $A(0)$ is given and normalized to one. Notice that, unlike Ngai and Pissarides (2007) , the TFP growth is still balanced across different sectors. Let δ denote the capital depreciation rate. When $K(t)<aL$,

$$\dot{K}=A(t)\frac{(\lambda-1)}{a}K(t)+A(t)L-\delta K(t)-C(t). \tag{37}$$

When $a^nL \le K(t)<a^{n+1}L$ for some $n \ge 1$, the capital accumulation function is

$$\dot{K}=A(t)\frac{\lambda^{n+1}-\lambda^n}{a^{n+1}-a^n}K(t)-\delta K(t)+\frac{\lambda^n(a-\lambda)}{a-1}A(t)L-C(t). \tag{38}$$

The social planner solves (19) subject to (37) and (38) with $K(0)$ taken as given. By following the same method as in Section 4, one can verify that sufficient conditions are satisfied to ensure that the equilibrium exists and is also unique. Solving the Hamiltonian yields the following Euler equation when $a^{n+1}L>K(t)\ge a^nL$ for any $n \ge 1$:

$$\frac{\dot{C}}{C}=\frac{A(t)\frac{\lambda^{n+1}-\lambda^n}{a^{n+1}-a^n}-\delta-\rho}{\sigma}, \tag{39}$$

which is a positive constant if and only if $A(t)(\lambda^{n+1}-\lambda^n)/(a^{n+1}-a^n)$ is a sufficiently large positive number independent of n. To satisfy this requirement, we assume

$$A(t)=\begin{cases} B\cdot\dfrac{a^n}{\lambda^n} & \text{if } t\in[t_n,t_{n+1}) \text{ for any } n\ge 1, \\ B\cdot\dfrac{a}{a-1} & \text{if } t\in[0,t_1) \end{cases} \tag{40}$$

where B is a positive constant and t_n is defined in Section 4. That is, the Hicks-neutral productivity $A(t)$ is a step function, which generically keeps constant and only jumps up discontinuously at the time point when a new industry just appears.[24] Then (39) becomes

$$\frac{\dot{C}}{C}=\frac{\frac{\lambda-1}{a-1}B-\delta-\rho}{\sigma}, \quad \forall t\in[0,\infty). \tag{41}$$

and $\dot{\eta}_{n,n+1}/\eta_{n,n+1}=\delta-(\lambda-1)/(a-1)B$. To ensure positive growth and also rule out explosive growth, we assume $((\lambda-1)/(a-1))B-\delta>(((\lambda-1)/(a-1))B-\delta-\rho)/\sigma>0$. Under those conditions, sustainable growth is achieved together with perpetual industry upgrading from labor-intensive industries to more and more capital-intensive industries step by step, because now the return to capital is sustained at a constant level $(\lambda-1)/(a-1)B$. Following the method in Section 4, one could also derive the endogenous capital accumulation equations sequentially (see Appendix 4).

However, this alternative standard setting is much less tractable than the setting in Section 4. In particular, the life-cycle dynamics of each industry no longer have a neat, closed-form characterization.[25] Moreover, to obtain sustainable growth and perpetual industry upgrading in this alternative setting, we must require the exogenous technology progress to have discontinuous jumps. This is because the functional form of the aggregate production function changes endogenously due to the endogenous structural change in the underlying disaggregate industry compositions. Nevertheless, the key qualitative features remain valid, including sustainable aggregate growth at a constant rate (41), perpetual industry upgrading

[23] The labor-augmenting exogenous technological progress does not yield perpetual industry upgrading for this case, although it generates sustainable aggregate growth. The intuition is the following: When the labor-augmenting technological progress is micro-founded at the industry level by entering into the Leontief industry production function, $K(t)/A(t)L$ for the aggregate economy will be a constant on the balanced growth path, so industrial upgrading also stops at some point. Please refer to Appendix 4.2 for more details.

[24] Note that without technological progress $F(K,L)$ is a piece-wise linear, continuous and concave function of K for any given L. Thus $A(t)$ must jump at each kink point of $F(K,L)$ to ensure that $A(t)F(K,L)$ has a constant return to K. The discontinuity of $A(t)$ could be avoided when the aggregate production function X is of general CES form with a positive and finite substitution elasticity, as is shown below.

[25] There are two main reasons that the setting in Section 4 is more tractable: (1) it gives us more linearity as it features the linear capital production of AK technology with a constant return to capital ξ, whereas the aggregate production function is only piece-wise linear and concave in capital (without technology progress) in the standard alternative setting. (2) The consumption–capital relation at the aggregate level derived from the static model (the bottom row of Table 6) can be directly applied to the dynamic analysis in the current setting, whereas in the more standard setting, the static model only yields the production–capital relation at the aggregate level instead of the consumption–capital relation, so the dynamic consumption analysis (Euler equation) becomes more complicated as its functional form is endogenously different over time without appropriately introduced technology progress.

(equivalent to $K(t)/L$ continuously and strictly increasing to infinity), hump-shaped dynamics of each industry, more capital-intensive industries reaching the peaks later, *etc.*

It is worth emphasizing that the perpetual hump-shaped industry dynamics (or structural change at the disaggregate sectors) are still driven by capital accumulation, which is in turn sustained by the (balanced) aggregate technology progress. So the theoretical mechanism for structural change is still different from non-homothetic preference or unbalanced TFP growth rates across sectors.

5.2. General CES function

A salient counterfactual feature of the previous analysis is that at any time point at most two industries coexist in equilibrium due to the infinite substitution elasticity across industries. Now we show that all the industries coexist when the aggregate production function is generalized to CES with finite substitution elasticity, but the key results remain valid, namely, that each industry still demonstrates a hump-shaped life-cycle pattern with more capital intensive industries reaching their peaks later. However, no closed-form solutions can be obtained due to the "curse of dimensionality" because no industries exit any more.

Assume everything remains the same as in Sections 3 and 4 except that now (5) is replaced by a more general CES function:

$$C = \left[\sum_{n=0}^{\infty} \lambda_n c_n^{\gamma} \right]^{1/\gamma}, \tag{42}$$

where $\gamma < 1$. We obtain the following result.

Proposition 4 (*Generalized Rybzynski Theorem*). *Suppose the production function is CES as defined in (42). All the industries will coexist in equilibrium. In addition, when capital endowment E increases, output and employment in industry n (for any $n \geq 1$) will both exhibit a hump-shaped pattern: an industry first expands with E and then declines with E after the industrial output reaches its peak. The more capital intensive an industry, the larger the capital endowment at which the industry reaches its peak.*

The proof is relegated to Appendix 5. Recall that the classical Rybczynski Theorem is silent about what happens when the number of goods exceeds two. In a model with N ($N > 2$) goods and two factors, as the country becomes more capital abundant, the existing theory does not give clear predictions on which sectors expand and which sectors shrink (Feenstra, 2003). Proposition 4 generalizes the Rybczynski Theorem by extending the commodity space from two-dimensional to infinite-dimensional, so one may call this proposition the Generalized Rybzynski Theorem.

For the dynamic analysis, we characterize the equilibrium in the same dynamic setting as in Section 4.[26] It is worth emphasizing that the hump-shaped dynamics and the timing fact also hold in the dynamic model even if all industries have the same productivity in final good production ($\lambda_n = 1$ for all n) when the aggregate production function is of CES with $\gamma < 1$ because now each industry must always exist at any finite price. So the only mechanism that drives industrial upgrading is still the change in the factor endowment because of homothetic preference and balanced productivity growth across sectors. More specifically, observe that

$$\frac{E}{C} = \frac{\sum_{n=1}^{\infty} a^n \left(\frac{\lambda^n}{1+a^n\theta} \right)^{1/(1-\gamma)}}{\left[1 + \sum_{n=1}^{\infty} \lambda^{n/(1-\gamma)} (1+a^n\theta)^{\gamma/(\gamma-1)} \right]^{1/\gamma}},$$

where θ denotes the endogenous rental-wage ratio, which turns out to be a strictly decreasing function of E as shown in the proof of Proposition 4 (see Appendix 5). So the above function implicitly determines $E \equiv G(C)$, where function $G(C)$ must satisfy $G'(C) > 0$ and $G''(C) \geq 0$. This is because the marginal productivity of capital in the final good production must be diminishing since the total labor supply is fixed while the aggregate production function is homogeneous of degree one with respect to capital and labor. Define $\psi(C) \equiv CG''(C)/G'(C)$, which is the elasticity of the change in marginal capital expenditure relative to the aggregate consumption change. For any specific time point t in the dynamic equilibrium, we must have the Euler equation: $\dot{C}(t)/C(t) = (\xi - \rho)/(\sigma + \psi(C(t)))$, which would degenerate into (26) when $\psi(C) = 0$, just as in the linear case ($\gamma = 1$).[27] This indicates that, as long as $C(t) > 0, E(t)$ must be strictly increasing over time in equilibrium (holding L fixed, we still assume $\xi - \rho > 0$ as before). Consequently, the dynamic path of the output of each individual industry must still exhibit a hump-shaped pattern in the full-fledged dynamic model, but it is no longer possible to obtain closed-form solutions for the entire dynamic path.

[26] Due to space limits, the equilibrium in the more standard alternative dynamic setting (that is, the one outlined in Section 5.1) is studied in Appendix 6. The key qualitative results turn out to be preserved.

[27] If $\lim \psi(C(t)) \to \infty$ as $C(t) \to \infty$, then the asymptotic aggregate consumption growth rate will be zero.

6. Conclusion

This paper explores the optimal industrial structure and the dynamics of each industry in a frictionless and deterministic economy with a highly tractable infinite-industry growth model. Closed-form solutions are obtained to fully characterize the endogenous process of repetitive hump-shaped industrial dynamics along the sustained growth path of the aggregate economy. The model generates all the features consistent with the stylized facts we observe in the data. We highlight the improvement in the endowment structure, which is given at any specific time but changeable over time (capital accumulation), as the fundamental driving force of the perpetual structural change at the disaggregated industry level in an economy. The model highlights the structural difference at different development levels by showing that the optimal industrial compositions are endogenously different as the endowment structure changes over time.

Several directions for future research seem particularly appealing. First, various market imperfections could be introduced into this first-best environment to see how industrial dynamics can be distorted by those frictions, which may allow for discussions on welfare-enhancing government policies. Second, it would be interesting to explore whether the mechanism highlighted in this paper can be extended to explain the dynamics of the sub-sectors in the service sector (see Buera and Kaboski, 2012a,b). Third, it may be fruitful to incorporate firm heterogeneity into each industry and study the entry, exit, and growth of firms within different industries together with industrial dynamics (see Hopenhayn, 1992; Luttmer, 2007; Rossi-Hansberg and Wright, 2007; Samaniego, 2010). Hopefully, the model developed in this paper can be useful to help us think further about all these issues.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2015.09.006.

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